

New
Specification



Rewarding Learning

**ADVANCED SUBSIDIARY (AS)
General Certificate of Education**

Further Mathematics

Assessment Unit AS 2

assessing

Applied Mathematics

[SFM21]

Assessment

**MARK
SCHEME**

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A

1. MW1 Correct resultant
 M1 Attempting scalar product
 W1 Correctly
 W1 Correct answer
- Notes
 [A] If use vector product, then MW1 only i.e. max of 1/4
 [B] If use product of magnitudes of \mathbf{F} , \mathbf{s} then MW1 only i.e. max of 1/4
2. (i) MW1 Correct normal reaction
 MW1 Correct weight
- Notes
 [A] Any additional forces then lose 1 mark
 [B] No arrows, lose each mark
- (ii) M1 Attempting to resolve vertically
 W1 Correct equation
 W1 Correct answer
- (iii) M1 Attempting to resolve horizontally
 M1 Using $F = ma$ with correct formula for acceleration
 W1 Correct LHS
 W1 Correct RHS
 W1 Correct radius
- Notes
 [A] Any extra forces included e.g. friction, then can award
 M1 M1 W0 W1 W0 i.e. max of 3/5
- (iv) MW1 Correct statement
3. (a) M1 Attempting to integrate
 W1 Correct limits
 MW1 Correct integrand
 W1 Correct expression
- (b) MW1 KEs zero
 MW1 Remaining energies at P correct
 M1 Attempting GPE using correct formula
 W1 Correct value
 M1 Attempting EPE using correct formula
 W1 Correct value
 M1 Using conservation of energy (ft their values for energies at P, Q)
 W1 PQ correct

4. (i) MW1 Speed conversion correct
 MW1 Force correct
 M1 Correct answer
- Notes
 [A] No conversion of speed – can award
 MW0 MW1 M0 i.e. max of 1/3
- (ii) M1 Attempting to resolve parallel to plane
 W1 Correctly
 M1 Trying to use Power = Force \times Velocity
 W1 Correct power
- (iii) M1 Attempting Newton's 2nd law
 W1 Correct $m \times a$
 W1 Correct forces
 W1 Correct propulsion force
 MW1 Correct velocity

5. (i) MW1 Force diagram correct

Notes

- [A] Weight must be given as 30g – not just W
 [B] No arrows on forces then MW0

- (ii) M1 Trying to use $Fr = \mu N$
 MW1 Correct normal reaction
 M1 Trying to use Work Done by Friction = $Fr \times$ displacement
 W1 Correct answer
- (iii) M1 Trying to use WD = Change in KE
 M1 Attempting difference of forces
 W1 LHS correct
 MW1 Initial KE correct
 MW1 Final KE correct
 W1 Correct answer

Section B

1. M1 Trying to use dimensions
 W1 Correct set-up
 M1 Trying to find dimensions – evidence of one correct
 W1 All dimensions correct
 M1 Knowing to compare powers
 W1 3 correct equations
 M1 Trying to solve equations
 W1 Correct answers

2. M1 Correctly combining relative velocities
W1 Directions of vectors correct
W1 Angles correct
M1 Trying to use sine rule
W1 Correctly substituted values
MW1 Correct angle
W1 Correct bearing
3. (i) M1 Trying to use Newton's law = weight on earth's surface
M1 Correct law of gravitation
W1 Correct answer
- (ii) W1 Correct change of units
M1 Using correct formula for period
W1 Correct ω
M1 Correct expression for central acceleration
MW1 Correct Newton's 2nd law
MW1 Correct r^3
W1 Correct r
W1 Correct height
4. (i) MW1 Correct diagram
- (ii) MW1 Correct extension
M1 Attempting correct expression for tension
W1 Correct answer
- (iii) MW1 Correct tension
M1 Trying to use $Fr = \mu N$
W1 Correct
MW1 Correct component of weight
M1 Trying to equate forces along plane
W1 Correct answer
5. (i) MW1 Correct vertical distance
MW1 Correct PE
- (ii) M1 Trying to use $E = KE + PE$
W1 Correct energy at C
MW1 Correct energy at A
M1 Trying to use conservation of energy
W1 Correct answer
- (iii) M1 Correct expression for acceleration
M1 Attempting Newton's 2nd law
W1 Correctly
W1 Correctly substitutes for mv^2 (allow ft of their answer from (ii))
W1 Correct N
- (iv) M1 Knowing reaction = 0
W1 Correct angle

Section C

1. (i) M1 Attempting to use formula for b
MW1 Correctly substituted
W1 Correct b
MW1 Correct \bar{x}
MW1 Correct \bar{y}
MW1 Correct a
W1 Correct equation
- (ii) M1 Attempting to find y
W1 Correct y
MW1 One valid comment
2. (i) MW1 Correct statement about randomly selecting clusters
MW1 Correct statement about random selection from each club
MW1 Correct statement referencing proportion selected
MW1 Correct statement about travelling costs or number of interview days
MW1 One valid comment
- (ii) MW1 One valid technique
MW1 One valid reason
3. (i) M1 Correct $\lambda = 2$
M1 Using $1 - P(X \leq 2)$
MW1 Correct terms used (or evidence of correct value from tables/calculator)
W1 Correct answer
- (ii) MW1 Correct substitution into correct formula
W1 Correct equation
M1 Attempting to use logs
W1 Correct answer
4. (i) M1 Trying to use combinations
W1 Correct answer
- (ii) MW1 1st correct product
MW1 2nd correct product
MW1 3rd correct product
W1 Correct answer
MW1 Correct probability
- (iii) MW1 Correct answer
- (iv) M1 Treating 2 juniors as a single item
W1 Obtaining $4!$
MW1 Correct answer

5. (i) M1 Trying to equate functions at $x = 0.5$
W1 Correct a
- (ii) MW1 Correct integral (integrand and limits)
MW1 Correct integration
W1 Correct answer
- (iii) MW1 Correct 0.125
M1 Correct integral
MW1 Correct limits
MW1 Correctly integrated
MW1 Correct substitutions
W1 Correct quadratic equation
M1 Attempting to solve quadratic
W1 2 correct solutions
W1 Correct answer selected
(if only correct answer stated then award final W2)

Alternative Solution

- (iii) MW1 Knowing area of trapezium = 0.125
M1 Trying to find area of trapezium
MW1 Using $(M - 0.5)$
MW1 Using $(-4M + \frac{17}{4})$
MW1 Expanding double brackets
Final 4 marks as in original solution

Section D

1. (i) M1 Attempting closed tour of vertices
W1 Stating every vertex correctly
- (ii) MW1 Edge from d (or g) identified
MW1 Correct edge
- (iii) M1 Attempting tour of edges
W1 1st half correct
W1 All correct
2. (i) M1 Attempting to form auxiliary equation
W1 Correctly
- (ii) MW1 Correctly solved quadratic
MW1 Forms expression correctly
M1 Attempts to equate 1st 2 terms
W1 Correctly
M1 Attempts to solve simultaneous equations
W1 Correct values
W1 Correct answer
- (iii) M1 Checks modulus of value < 1
W1 States tends to zero

3. (i) MW1 Correct answer
- (ii) MW1 E1, E2, E4, E5, E7 correct
 MW1 E3, E6, E8 correct
 MW1 L8, L6, L5 correct
 MW1 Other Ls correct
- (iii) (a) M1 Correct answer
 (b) M1 Tries to subtract early from late time
 W1 Correct answer
- (iv) MW1 Correct critical path
 MW1 Correct answer
4. M1 Attempts set up for 1st 3 columns
 W1 Correct
 MW1 Columns 4, 5, 6 correct
 MW5 1 mark for each of columns 7 – 11 correct
 MW1 States column 9 = column 11
5. (i) MW1 Compares 2 suitable products
 M1 States not commutative
- (ii) MW1 Period of identity and two order 2 elements correct
 MW1 Other order 2 elements correct
 MW1 Order 4 elements correct
- (iii) MW2 1 mark for each subgroup
- (iv) MW4 1 mark for each element and period correct
- (v) MW1 Identify correct subgroup
 M1 Correct reason

SECTION A: Mechanics 1

AVAILABLE
MARKS

1 Resultant = $\begin{pmatrix} 5 \\ 6 \\ -7 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$

MW1

Work done = $\mathbf{F} \cdot \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$

M1 W1

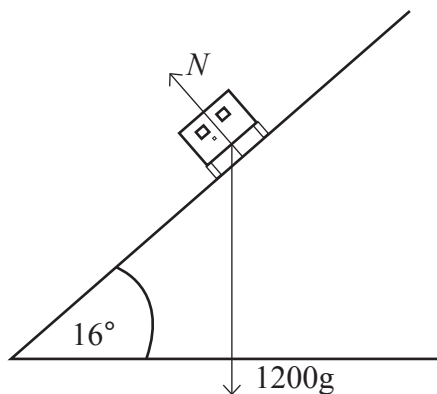
= -18J

W1

4

2 (i)

MW2



(ii) $N \cos 16^\circ = 1200 \times 9.8$
 $N = 12233.9 = 12200 \text{ N (3 s.f.)}$

M1 W1
W1

(iii) $N \sin \theta = \frac{mv^2}{r}$

M2

$N \sin 16^\circ = \frac{1200 \times 14^2}{r}$

W2

$r = 69.7 \text{ m (3 s.f.)}$

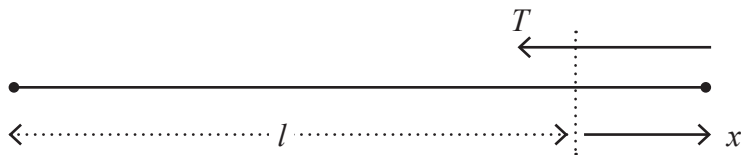
W1

(iv) Treat the car as a particle.

MW1

11

3 (a)



$$\text{Work done by tension} = \int_0^e -T \, dx$$

$$= \int_0^e -\left(\frac{\lambda x}{l}\right) dx$$

$$= -\frac{\lambda e^2}{2l} \text{ J}$$

M1 W1

MW1

W1

(b) KE at P = 0 and KE at Q = 0

MW1

When at P:

Gravitational PE = 0

Elastic PE = 0

MW1

When at Q:

Gravitational PE = $-mgh = -5 \times 9.8 \times (1.7 + e)$

M1 W1

$$\text{Elastic PE} = \frac{\lambda e^2}{2l} = \frac{98 \times e^2}{2 \times 1.7}$$

M1 W1

By Conservation of Mechanical Energy:

Total energy at P = Total energy at Q

M1

$$0 = \frac{49e^2}{1.7} - 49(1.7 + e)$$

$$e^2 - 1.7e - 2.89 = 0$$

$$e = 2.75 \text{ m}$$

$$\text{PQ} = 4.45 \text{ m (3 s.f.)}$$

W1

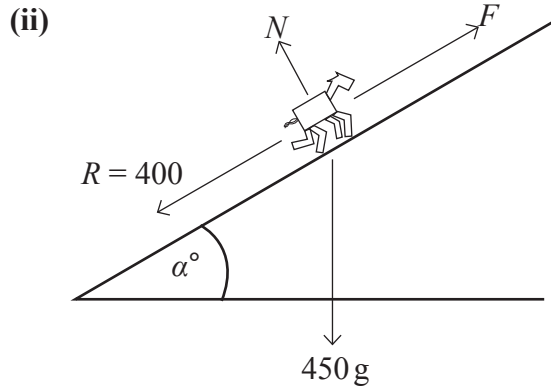
AVAILABLE
MARKS

12

4 Running force = F
Resistance = R

- (i) Speed = $18/3.6 = 5 \text{ m s}^{-1}$
Running force $F = 2000/5 = 400 \text{ N}$
Constant velocity $\Rightarrow R = F = 400 \text{ N}$

MW1
MW1
M1



$$F = R + 450 \text{ g} \sin \alpha$$

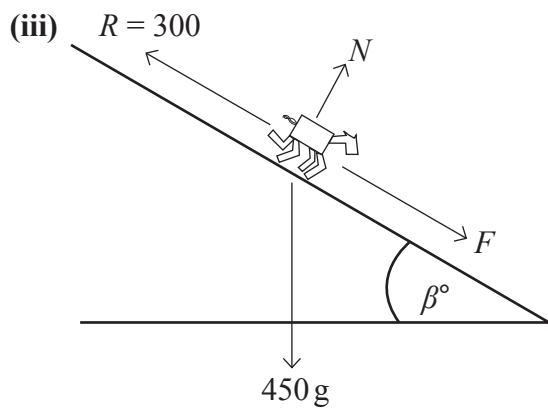
$$\text{Speed} = 10.8/3.6 = 3 \text{ m s}^{-1}$$

$$\text{Power} = 3(400 + 45 \times 9.8)$$

$$\text{Power} = 2523 \text{ W} \rightarrow 2520 \text{ W (3 s.f.)}$$

M1 W1

M1
W1



$$450 \times 0.4 = 450 \text{ g} \times 0.05 - 300 + F$$

$$F = 259.5 \text{ N}$$

$$v = 2000/259.5 = 7.707 \text{ m s}^{-1}$$

$$v = 7.71 \text{ m s}^{-1} \text{ (3 s.f.)}$$

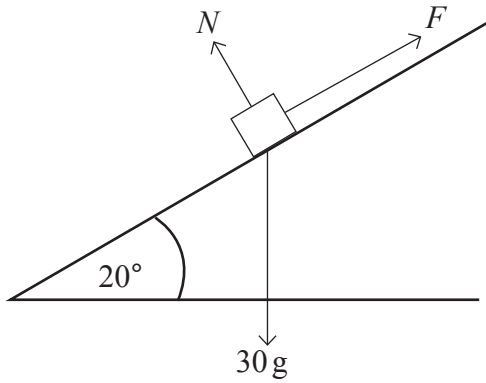
M1 W2
W1

MW1

AVAILABLE
MARKS

12

5 (i)



MW1

AVAILABLE
MARKS

(ii) Friction = μN

M1

$$N = mg \cos \alpha = 30 \times 9.8 \times \cos 20^\circ$$

MW1

Work done by friction = force \times displacement

M1

$$= -0.3 \times 30 \times 9.8 \times \cos 20^\circ \times 4$$

$$= -331.524$$

$$= -332 \text{ J (3 s.f.)}$$

W1

(iii) Work done by resultant force = change in kinetic energy

M1

$$mg \sin 20^\circ \times 4 - 331.5 = \frac{1}{2}mv^2 - \frac{1}{2}m \times 0.8^2$$

M1 W1 MW2

$$30 \times 9.8 \times \sin 20^\circ \times 4 - 331.5 = \frac{1}{2} \times 30 \times v^2 - \frac{1}{2} \times 30 \times 0.8^2$$

$$v = 2.31 \text{ ms}^{-1} \text{ (3 s.f.)}$$

W1

11

Section A

50

SECTION B: Mechanics 2

AVAILABLE MARKS

1 $M = k \mu^x \rho^y H^z$

Taking dimensions

$$M = (ML^{-1}T^{-1})^x (ML^{-3})^y (ML^2T^{-3})^z$$

Equating indices:

M: $1 = x + y + z$ (1)

L: $0 = -x - 3y + 2z$ (2)

T: $0 = -x - 3z$ (3)

Thus $x = -3z$

$3(1) + (2)$ gives $3 = 2x + 5z$

Thus $z = -3$ and $x = 9$

$y = 1 - x - z = -5$

$M = k \mu^9 \rho^{-5} H^{-3}$

M1 W1

M1 W1

M1

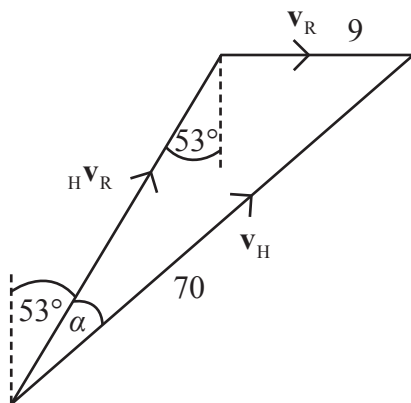
W1

M1

W1

8

2



M1 W1 W1

$$\frac{\sin \alpha}{9} = \frac{\sin 143^\circ}{70}$$

M1 W1

$$\alpha = \sin^{-1} \left(\frac{9 \times \sin 143^\circ}{70} \right) = 4.4378^\circ$$

MW1

Bearing = 057° (= N 57.4° E)

W1

7

3 (i) Let the mass of the satellite be m kg

$$\text{On the earth's surface } mg = \frac{mMG}{R^2}$$

M2

$$MG = gR^2 = 9.8 \times 6378000^2 = 3.9865 \times 10^{14} \text{ m}^3\text{s}^{-2}$$

W1

(ii) Let r metres be the radius of the satellite's orbit.

Angular velocity of the satellite = ω

Period of the satellite in seconds = $90 \times 60 = 5400$ s

W1

Thus $5400 = \frac{2\pi}{\omega}$ where ω is the angular velocity of the earth

M1

So $\omega = \frac{\pi}{2700}$ radians per second

W1

Central acceleration of the satellite = $\omega^2 r$

M1

By Newton's 2nd Law

$$m\omega^2 r = \frac{mMG}{r^2}$$

MW1

$$r^3 = \frac{MG}{\omega^2}$$

MW1

$$r = 6653 \text{ km}$$

W1

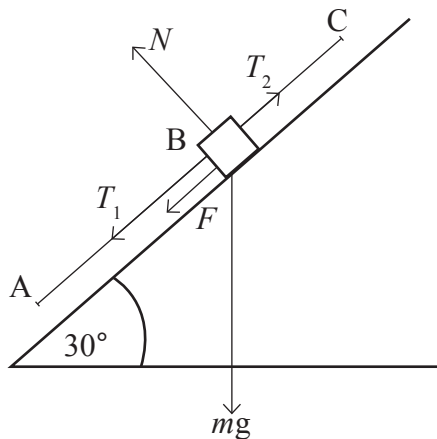
Height above the earth's surface = $6653 - 6378 = 275$ km

W1

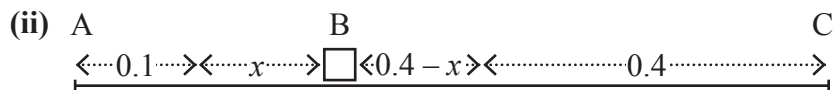
AVAILABLE
MARKS

11

4 (i)



MW1



Extension in BC = $0.9 - 0.1 - 0.4 - x = 0.4 - x$

MW1

Tension in BC = $k_2 mg \frac{(0.4 - x)}{0.4} = \frac{1}{2} k_2 mg (2 - 5x)$

M1 W1

(iii) Tension in AB = $k_1 mg \frac{x}{0.1} = 10k_1 mgx$

MW1

Friction = $\mu mg \cos 30^\circ = \frac{1}{\sqrt{3}} mg \cos 30^\circ = \frac{1}{2} mg$

M1 W1

Component of weight along plane = $mg \sin 30^\circ = \frac{1}{2} mg$

MW1

$10k_1 mgx + \frac{1}{2} mg + \frac{1}{2} mg = \frac{1}{2} k_2 mg (2 - 5x)$

M1

$10k_1 mgx + \frac{5}{2} k_2 mgx = k_2 mg - mg$

$10k_1 x + \frac{5}{2} k_2 x = k_2 - 1$

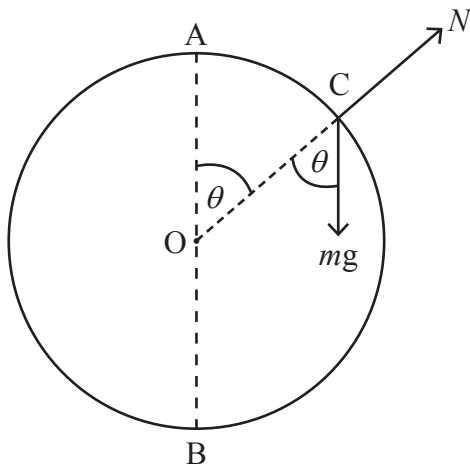
$x = \frac{2(k_2 - 1)}{5(4k_1 + k_2)}$

W1

10

AVAILABLE MARKS

5 (i)



Vertical drop from C to B = $r + r \cos \theta$

MW1

Potential energy at C = $mgr(1 + \cos \theta)$

MW1

(ii) Energy of car at C = $\text{KE} + mgr(1 + \cos \theta)$

M1 W1

Energy of car at A = $0 + 2mgr$

MW1

$\text{KE} = mgr(1 - \cos \theta)$

M1 W1

(iii) Expression for radial acceleration = $\frac{mv^2}{r}$

M1

Newton's 2nd Law

$$\frac{mv^2}{r} = mg \cos \theta - N$$

M1 W1

Using (ii) $2mg(1 - \cos \theta) = mg \cos \theta - N$

W1

$N = mg(3 \cos \theta - 2)$ newtons

W1

(iv) Loss of contact from drum $\Rightarrow N = 0$

M1

$$\cos \theta = \frac{2}{3}$$

$\theta = 48.2^\circ$ (3 s.f.)

W1

AVAILABLE
MARKS

Section B

14

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SECTION C: Statistics

AVAILABLE
MARKS

1 (i) $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$ M1

$b = \frac{382.6 - \frac{100 \times 39.1}{8}}{1610 - \frac{100^2}{8}}$ MW1

$b = -\frac{2123}{7200} = -0.295$ (3 s.f.) W1

$\bar{x} = \frac{\Sigma x}{n} = \frac{100}{8} = 12.5$ MW1

$\bar{y} = \frac{\Sigma y}{n} = \frac{39.1}{8} = 4.8875$ MW1

$a = \bar{y} - b\bar{x} = 4.8875 + \frac{2123}{7200}(12.5) = 8.57$ (3 s.f.) MW1

$y = 8.57 - 0.295x$ (3 s.f.) W1

(ii) When $x = 30$ $y = a + b(30)$ M1
 $y = -0.273$ mm W1

Negative value for tread depth is not possible. Impossible value for y may arise from data that deviates at large distance from the linear relationship. MW1

OR

Negative value for tread depth is not possible. Values estimated by extrapolation can be unreliable. MW1

10

			AVAILABLE MARKS		
2	(i) Treat each hockey club as a cluster. Randomly select a pre-decided number of clusters (clubs). Decide on the sample size and on the proportion of supporters to be selected from each chosen club. Take a random sample of supporters from each club selected, in proportion to the relative size of the club.	MW1	7		
		MW1			
		MW1			
	Advantage: Interviews only need to be conducted at a sample of clubs. This reduces travelling costs and the number of days required for interviews, if, for example, all the interviews from a particular club can be scheduled for a single day.			MW1	
	Disadvantage: There will be similarities between supporters within a given club. Therefore, the range of views in the sample may not be representative of the full range of views in the population.			MW1	
	(ii) Answer: Simple random sampling Reason: The number of clubs represented in the sample is likely to be greater when compared with cluster sampling. Therefore, there will be a greater cost associated with visiting clubs to conduct interviews. There will also be a cost associated with constructing the complete sampling frame.	MW1			
		OR			
		Answer: Stratified sampling, where each club forms a separate stratum		MW1	
		Reason: Each club (stratum) would need to be visited to conduct interviews, increasing the cost compared with cluster sampling. There will also be a cost associated with constructing the complete sampling frame.		MW1	
				MW1	
3	(i) X is the random variable ‘number of earthquakes of magnitude 8.0 or greater per year’ $X \sim \text{Po}(2)$	M1	8		
		$P(X > 2) = 1 - P(X \leq 2)$		M1	
		$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$		MW1	
	$= 1 - e^{-2} \left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} \right)$			MW1	
	$= 0.323$ (3 s.f.)			W1	
	(ii) Y is the random variable ‘number of earthquakes of magnitude 9.0 or greater per year’ $Y \sim \text{Po}(\lambda)$	$P(Y = 0) = \frac{e^{-\lambda} \lambda^0}{0!}$		MW1	
		$0.25 = e^{-\lambda}$		W1	
		$-\ln 0.25 = \lambda$		M1	
		$\lambda = 1.39$ (3 s.f.)		W1	
				W1	

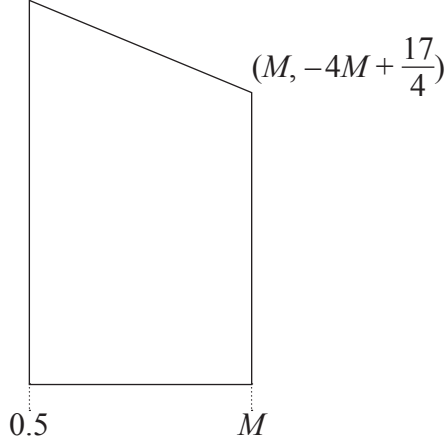
		AVAILABLE MARKS
4	<p>(i) ${}^{13}C_5 = 1287$</p> <p>(ii) (5S and 0J) or (4S and 1J) or (3S and 2J) $= ({}^6C_5 \times {}^7C_0) + ({}^6C_4 \times {}^7C_1) + ({}^6C_3 \times {}^7C_2)$ $= 531$</p> <p>$P(S > J) = \frac{531}{1287} = \frac{59}{143} = 0.413$ (3 s.f.)</p> <p>(iii) $5! = 120$</p> <p>(iv) Treating two juniors as a single item gives $4!$ arrangements The two juniors can be arranged in two different ways, so the total number of arrangements $= 2 \times 4!$ $= 48$</p>	<p>M1 W1</p> <p>MW3 W1</p> <p>MW1</p> <p>MW1</p> <p>M1 W1</p> <p>MW1</p>
5	<p>(i) $9(0.5)^2 = a(0.5) + \frac{17}{4}$</p> <p>$a = -4$</p> <p>(ii) $P(0 < X < 0.5) = \int_0^{0.5} 9x^2 dx$</p> <p>$= 3[x^3]_0^{0.5}$</p> <p>$= 3(0.5^3 - 0^3)$</p> <p>$= 0.375$</p> <p>(iii) $0.5 - 0.375 = 0.125$</p> <p>$\int_{0.5}^M -4x + \frac{17}{4} dx = 0.125$</p> <p>$\left[-2x^2 + \frac{17}{4}x\right]_{0.5}^M = 0.125$</p> <p>$-2M^2 + \frac{17}{4}M - \left[-2(0.5)^2 + \frac{17}{4}(0.5)\right] = 0.125$</p> <p>$2M^2 - \frac{17}{4}M + \frac{7}{4} = 0$</p> <p>$8M^2 - 17M + 7 = 0$</p> <p>$M = \frac{17 \pm \sqrt{65}}{16}$</p> <p>$M_1 = 1.57$ ($> 1 \therefore$ ignore) $M_2 = 0.559$</p> <p>Median = 0.559 (3 s.f.)</p>	<p>M1</p> <p>W1</p> <p>MW1</p> <p>MW1</p> <p>W1</p> <p>MW1</p> <p>M1 MW1</p> <p>MW1</p> <p>MW1</p> <p>MW1</p> <p>W1</p> <p>M1</p> <p>W1</p> <p>W1</p>
<p>Alternative solution for part (iii) is given overleaf on page 20</p>		
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Alternative solution for part (iii)

$$0.5 - 0.375 = 0.125$$

$$\text{Area of trapezium} = 0.125$$

(0.5, 2.25)



$$0.125 = \frac{1}{2} \left(2.25 + \frac{17}{4} - 4M \right) (M - 0.5)$$

$$0.25 = 6.5M - 3.25 - 4M^2 + 2M$$

$$8M^2 - 17M + 7 = 0$$

$$M = \frac{17 \pm \sqrt{65}}{16}$$

$$M_1 = 1.57 (> 1 \therefore \text{ignore}) \quad M_2 = 0.559$$

$$\text{Median} = 0.559 \text{ (3 s.f.)}$$

MW1

M1 MW2

MW1

W1

M1

W1

W1

AVAILABLE
MARKS

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Section C

50

SECTION D: Discrete and Decision Mathematics

AVAILABLE
MARKS

- 1 (i) a e b c d f g h i a include all vertices M1
any correct cycle W1
- (ii) edge from vertex d to vertex g MW2
- (iii) e b c d i a e c f d g i h g f h e M1
16 edges W1
outside edges W1
inside edges
(last 2 marks for any correct circuit)
(need attempt at a full circuit for any marks)
- 2 (i) Substitute $L_n = kx^n$
obtaining $kx^{n+2} = kx^{n+1} + kx^n$ M1
 $kx^n(x^2 - x - 1) = 0$
 $x^2 - x - 1 = 0$ or $x = 0$ W1
- (ii) $x^2 - x - 1 = 0$
$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$
 MW1
$$L_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$
 MW1
 $L_0 = 2 = A + B$
and $L_1 = 1 = \frac{A}{2}(1+\sqrt{5}) + \frac{B}{2}(1-\sqrt{5})$ M1 W1
substituting $B = 2 - A$ and solving }
$$1 = \frac{A}{2}[(1+\sqrt{5}) - (1-\sqrt{5})] + 1 - \sqrt{5}$$
 M1
 $A = 1, B = 1$ W1
Thus $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$ W1
- (iii) $\left|\frac{1-\sqrt{5}}{2}\right| = |-0.618| < 1$ M1
 $\left(\frac{1-\sqrt{5}}{2}\right)^n \rightarrow 0$ as $n \rightarrow \infty$
 $\therefore L_n \rightarrow 2^n(1+\sqrt{5})^n$ W1

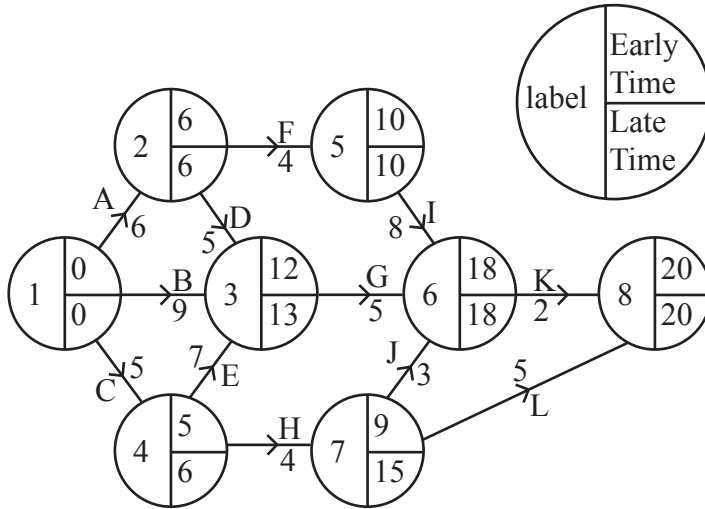
7

11

- 3 (i) Early time of a node is the greatest of the set of early times of preceding nodes plus the length of intervening activities
Thus early time of node 3 = $\max(6 + 5, 9, 5 + 7) = 12$

MW1

- (ii) Key for event nodes



- E_i = early time at node i
 L_i = late time at node i
 $E_1 E_2 E_5 E_4 E_7$
 $E_3 E_6 E_8$
 $L_8 L_6 L_5$
 rest of L_i s

MW1
 MW1
 MW1
 MW1

- (iii) (a) Float time for activity H is the maximum number of days after the end of the preceding activity C by which event H must be started, to be finished in time

M1

- (b) Float time for H = $15 - 5 - 4 = 6$ days

M1 W1

- (iv) Critical activities A F I K
 Length 20 days

MW1
 MW1

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4

p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p$ or r	$(\sim p$ or r) and q	$\sim[(\sim p$ or $r)$ and $q]$	p and $\sim r$	$(p$ and $\sim r)$ or $\sim q$
T	T	T	F	F	F	T	T	F	F	F
T	T	F	F	F	T	F	F	T	T	T
T	F	T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	F	F	T	T	T
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	T	F	F	F
F	F	T	T	T	F	T	F	T	F	T
F	F	F	T	T	T	T	F	T	F	T

Column 9

Column 11

- Set up for first 3 columns
 Columns 4, 5, 6
 Columns 7, 8, 9, 10, 11
 Stating column 9 = column 11

M1 W1
 MW1
 MW5
 M1

9

- 5 (i) $ba = a^3b \neq ab$
so not commutative

MW1
M1

(ii)

Element	1	a	a ²	a ³	b	ab	a ² b	a ³ b
Period	1	4	2	4	2	2	2	2

MW3

- (iii) (1, a, a², a³) and (1, a², b, a²b)

MW2

(iv)

Element	Period
Identity	1
Rotation of 180° about centre	2
Reflection in line PT	2
Reflection in line RV	2

MW4

- (v) Isomorphic to the following subgroup of G

Element	Period
1	1
a ²	2
b	2
a ² b	2

MW1

Reason – Identical periods or a correct isomorphism

M1

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Section D

50

Total

100

AVAILABLE
MARKS